1

Real Numbers

Memory Map

Real Numbers

Theorems

1. Let p be a prime number. If p divides a^2, then p divides a, where a is positive integer.

2. If p is prime, $\sqrt{p}$ is irrational $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}$ etc. are irrational.

Prime Factorisation Method

For any two positive integers, HCF $(a, b) \times$ LCM $(a, b) = a \times b$.

For example :

$a = pq^2$

$b = p^3q$

HCF $= pq$

LCM $= p^3q^2$

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Composite Number $x = p_1 \times p_2 \times p_3 \times .... \times p_n$, where $p_1, p_2, ... p_n$ are prime numbers.

Competency Based Questions

Multiple Choice Questions

1. L.C.M. (pq^2, p^3q), where p, q are prime is
   (a) pq (b) p^3q^3
   (c) p^3q^2 (d) p^2q^2
2. H.C.F. (pq^2, p^3q), where p, q are prime is
   (a) pq (b) p^3q^3
   (c) p^3q^2 (d) p^2q^2
3. H.C.F. $(p^3q^2, pq^3) =$
   (a) pq (b) pq^2
   (c) p^3q^3 (d) p^3q^2
4. The exponent of 2 in the prime factorization of 144, is
   (a) 3 (b) 4
   (c) 5 (d) 6
5. The sum of the exponents in the prime factorisation of 196, is
   (a) 1 (b) 2
   (c) 4 (d) 6
6. If L.C.M. $(a, 18) = 36$ and H.C.F. $(a, 18) = 2$, then $a =$
   (a) 2 (b) 3
   (c) 4 (d) 1
7. If H.C.F. $(26, 169) = 13$, then L.C.M. $(26, 169)$ = ...
   (a) 26 (b) 52
   (c) 338 (d) 13
8. H.C.F. (65, 117) is equal to
   (a) 5 (b) 9
   (c) 13 (d) None of these
9. If H.C.F. $(26, 169) = x$ and L.C.M. $(26, 169) = 338$, then $x =$
   (a) 26 (b) 52
   (c) 338 (d) 13
10. If p and q are co-prime numbers, then p^2 and q^2 are
    (a) co-prime (b) not co-prime
    (c) even (d) odd
11. If the L.C.M. of 4 and 18 is 36 and H.C.F. of 4 and 18 is x, then $x =$
    (a) 2 (b) 3
    (c) 4 (d) 1
12. The L.C.M. of two numbers is 1200. Which of the following cannot be their H.C.F. ?
    (a) 600 (b) 500
    (c) 400 (d) 200
13. If $a = pq^4$ and $b = p^2q$ (p, q are primes), then L.C.M. (a and b) =
    (a) pq (b) p^3q^3
    (c) p^2q^4 (d) p^2q^2
14. H.C.F. (95, 152) =
    (a) 57 (b) 1
    (c) 19 (d) 38.
15. If H.C.F. of 26 and 91 is 13, then L.C.M. of 26 and 91 is
    (a) 91 (b) 182
    (c) 78 (d) 104 **(C.B.S.E. 2010)**
16. If p, q are two consecutive natural numbers, then H.C.F. (p, q) is
    (a) q (b) p
    (c) 1 (d) pq
    (C.B.S.E. 2010, 2013)
17. If $xy = 180$ and H.C.F. $(x, y) = 5$, then L.C.M. (x, y) is
    (a) 90 (b) 18
    (c) 36 (d) 5 **(C.B.S.E. 2018)**
18. If p, q are two prime numbers, then L.C.M. (p, q) is
    (a) 1 (b) p
    (c) q (d) pq **(C.B.S.E. 2010)**
19. If d = L.C.M. (36, 198), then the value of d is
    (a) 396 (b) 198
    (c) 36 (d) 1 **(C.B.S.E. 2010)**
20. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is
    (a) xy (b) xy^2
    (c) x^3y^3 (d) x^2y^2
    (N.C.E.R.T. Exemplar)

21. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is

(*a*) ab (*b*) a^2b^2

(*c*) a^3b^2 (*d*) a^3b^3

(N.C.E.R.T. Exemplar)

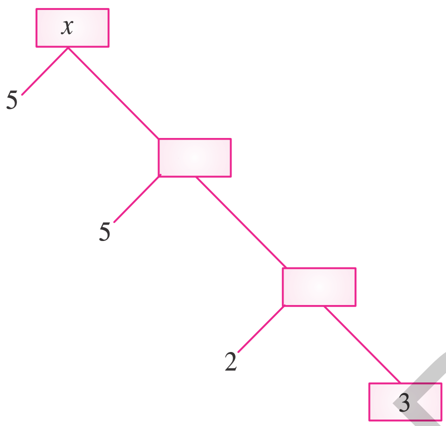
22. The (H.C.F. × L.C.M.) for the numbers 50 and 20 is

(*a*) 10 (*b*) 100

(*c*) 1000 (*d*) 50

(C.B.S.E. 2010, Term-I, 2021-22)

23. The value of x in the factor tree is



(*a*) 30 (*b*) 150

(*c*) 100 (*d*) 50

24. If d = H.C.F. (48, 72), the value of d is

(*a*) 24 (*b*) 48

(*c*) 12 (*d*) 72

(C.B.S.E. 2010, 2011)

25. H.C.F. of two consecutive even numbers is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 4

(C.B.S.E. 2011, 2012, Term-I, 2021-22)

26. $(-1)^n + (-1)^{8n} = 0$, where n is :

(*a*) any +ve integer

(*b*) any odd natural number

(*c*) any even natural number

(*d*) any –ve integer. *(C.B.S.E. 2012)*

27. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM $(a, b, c) = 2^3 \times 3^2 \times 5$, then n =

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

28. Which of the following is rational ?

(*a*) $\sqrt{6} + \sqrt{9}$ (*b*) $\sqrt{2} + \sqrt{4}$

(*c*) $\sqrt{4} + \sqrt{9}$ (*d*) $\sqrt{3} + \sqrt{5}$

29. $(2 + \sqrt{5})(2 + \sqrt{5})$ is :

(*a*) a rational number

(*b*) a whole number

(*c*) an irrational number

(*d*) a natural number *(C.B.S.E. 2010)*

30. $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$ is :

(*a*) a rational number

(*b*) a whole number

(*c*) an irrational number

(*d*) a natural number *(C.B.S.E. 2010)*

31. If H.C.F. of 65 and 117 is expressible in the form $65\,m - 117$, then the value of m is

(*a*) 4 (*b*) 2

(*c*) 1 (*d*) 3

(N.C.E.R.T. Exemplar)

32. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is

(*a*) 13 (*b*) 65

(*c*) 875 (*d*) 1750

(N.C.E.R.T. Exemplar)

33. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(*a*) 10 (*b*) 100

(*c*) 504 (*d*) 2520

(N.C.E.R.T. Exemplar)

34. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer (*b*) a natural number

(*c*) an odd integer (*d*) an even integer

(N.C.E.R.T. Exemplar)

35. Which of the following is a pair of co-primes ?

(*a*) (14, 35) (*b*) (18, 25)

(*c*) (31, 93) (*d*) (32, 62)

(C.B.S.E. Term-I, 2021-22)

36. The exponent of 5 in the prime factorisation of 3750 is :

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

(C.B.S.E. Term-I, 2021-22)

37. What is the greatest possible speed at which a girl can walk 95 m and 171 m in an exact number of minutes ?
(*a*) 17 m/min (*b*) 19 m/min
(*c*) 23 m/min (*d*) 13 m/min
(C.B.S.E. Term-I, 2021-22)

38. For which natural number n, 6^n end with digit zero ?
(*a*) 6 (*b*) 5
(*c*) 0 (*d*) None of these
(C.B.S.E. 2026)

39. HCF of 92 and 152 is :
(*a*) 4 (*b*) 19
(*c*) 23 (*d*) 57
(C.B.S.E. Term-I, 2021-22)

40. Three alarm clocks ring their alarms at regular interval of 20 min., 25 min. and 30 min. respectively. If they first beep together at 12 noon, at what time will they again beep for the first time ?
(*a*) 4 : 00 pm (*b*) 4 : 30 pm
(*c*) 5 : 00 pm (*d*) 5 : 30 pm
(C.B.S.E. Term-I, 2021-22)

41. If a and b are two co-prime numbers, then a^3 and b^3 are :
(*a*) co-prime (*b*) not co-prime
(*c*) even (*d*) odd
(C.B.S.E. Term-I, 2021-22)

42. HCF and LCM of 12, 21, 15 respectively are :
(*a*) 3, 140 (*b*) 12, 420
(*c*) 3, 420 (*d*) 420, 3
(C.B.S.E. 2020, Term-I, 2021-22)

43. If both 525 and 3000 are divisible only by 3, 5, 15, 25 and 75, then HCF (525, 3000) =
(*a*) 25 (*b*) 125
(*c*) 75 (*d*) 15
(C.B.S.E. Term-I, 2021-22)

44. The prime factorisation of natural number 288 is
(*a*) $2^4 \times 3^3$ (*b*) $2^4 \times 3^2$
(*c*) $2^5 \times 3^2$ (*d*) $2^5 \times 3^1$ **(C.B.S.E. 2023)**

45. The HCF of two numbers 65 and 104 is 13. If LCM of 65 and 104 is $40x$, the value of x is :
(*a*) 5 (*b*) 13
(*c*) 40 (*d*) 8 **(C.B.S.E. 2024)**

46. The L.C.M. of 960 and 240 is :
(*a*) 960 (*b*) 240
(*c*) 60 (*d*) 15 **(CBSE 2026)**

47. The natural number 1 is :
(*a*) a prime number
(*b*) a composite number
(*c*) prime as well as composite
(*d*) neither prime nor composite **(CBSE 2026)**

48. For any natural number n, 5^n ends with the digit :
(*a*) 0 (*b*) 5
(*c*) 3 (*d*) 2 **(CBSE 2026)**

49. $7 \times 11 \times 13 + 5$ is
(*a*) a prime number
(*b*) an odd number
(*c*) a composite number
(*d*) a multiple of 5 **(CBSE 2026) (Basic)**

Competency Based Questions

Answers

1. (*c*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*a*)	**11.** (*a*)	**12.** (*b*)
13. (*c*)	**14.** (*c*)	**15.** (*b*)	**16.** (*c*)	**17.** (*c*)	**18.** (*d*)	**19.** (*a*)	**20.** (*b*)	**21.** (*c*)	**22.** (*c*)	**23.** (*b*)	**24.** (*a*)
25. (*c*)	**26.** (*b*)	**27.** (*b*)	**28.** (*c*)	**29.** (*c*)	**30.** (*a*)	**31.** (*b*)	**32.** (*a*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*b*)
37. (*b*)	**38.** (*d*)	**39.** (*a*)	**40.** (*c*)	**41.** (*a*)	**42.** (*c*)	**43.** (*c*)	**44.** (*c*)	**45.** (*b*)	**46.** (*a*)	**47.** (*d*)	**48.** (*b*)
49. (*c*)											

Hints & Explanations

1. (*c*) [$\because$ L.C.M. $(pq^2, p^3q) = p^3q^2$]

2. (*a*) [H.C.F. $(pq^2, p^3q) = pq$]

3. (*b*) [H.C.F. $(p^3q^2, pq^3) = pq^2$]

4. (*b*) [$\because$ $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 2^4 \times 3^2$ $\therefore$ exponent of 2 = 4]

5. (*c*) [$\because$ $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$
$\therefore$ sum of exponents = 2 + 2 = 4]

6. (*c*) [$\because$ L.C.M. $\times$ H.C.F. = product of two numbers $\Rightarrow 36 \times 2 = a \times 18 \Rightarrow a = 4$]

7. (*c*) [H.C.F. $\times$ L.C.M. $= 26 \times 169 \Rightarrow 13 \times$ L.C.M. $= 26 \times 169 \Rightarrow$ L.C.M. $= 2 \times 169 = 338$]

8. (*c*) [$\because$ $65 = 5 \times 13$; $117 = 9 \times 13$
$\therefore$ H.C.F. = 13]

9. (*d*) [$\because$ $x \times 338 = 26 \times 169$

$$\Rightarrow x = \frac{26 \times 169}{338} = 13]$$

10. (*a*) [$\because$ p and q are co-prime numbers, p^2, q^2 are also co-prime.]

11. (*a*) [$\because$ $4 \times 18 = 36 \times x \Rightarrow x = 2$]

12. (*b*) [Since L.C.M. is a multiple of H.C.F. and 1200 is not a multiple of 500]

13. (*c*) [L.C.M. $(a, b) =$ L.C.M. $(pq^4, p^2q) = p^2q^4$]

14. (*c*) [$95 = 5 \times 19$; $152 = 2 \times 2 \times 2 \times 19$

$= 2^3 \times 19$ $\therefore$ H.C.F. $= 19$]

15. (*b*) [H.C.F. $\times$ L.C.M. $= 26 \times 91$

$\Rightarrow 13 \times$ L.C.M. $= 26 \times 91$

$\Rightarrow$ L.C.M. $= \dfrac{26 \times 91}{13} = 182$]

16. (*c*) [H.C.F. $(p, q) = 1$ ($\because$ p, q are two consecutive natural numbers)

$\therefore$ There is no common factor between p and q.]

17. (*c*) [(L.C.M.) (H.C.F.) $= xy = 180$

$\Rightarrow$ L.C.M. $= \dfrac{180}{\text{H.C.F.}} = \dfrac{180}{5} = 36$]

Note : L.C.M. must be a multiple of H.C.F. (Ignoring as there is no option correct.)

18. (*d*) [Since p, q are two prime numbers $\therefore$ There is no common factor between p and q.

$\therefore$ L.C.M. $(p, q) = pq$

19. (*a*) [$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$;

$198 = 2 \times 3 \times 3 \times 11 = 2 \times 3^2 \times 11$

L.C.M. $= 2^2 \times 3^2 \times 11 = 4 \times 9 \times 11$

$= 36 \times 11 = 396$]

20. (*b*) [H.C.F. of $(a, b) =$ H.C.F. $(x^3y^2, xy^3) = xy^2$]

21. (*c*) [L.C.M. $(p, q) =$ L.C.M. $(ab^2, a^3b) = a^3\, b^2$]

22. (*c*) [H.C.F. $\times$ L.C.M. $=$ product of two numbers $= 50 \times 20 = 1000$]

23. (*b*) [Second rectangle from bottom has value $= 2 \times 3 = 6$)

Third rectangle has value $= 5 \times 6 = 30$

$\therefore$ $x = 5 \times 30 = 150$]

24. (*a*) [$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$;

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

$\therefore$ H.C.F. $(48, 72) = 2^3 \times 3 = 24$]

25. (*c*) [holds clearly]

26. (*b*) [$\because$ $(-1)^{\text{odd}} + (-1)^{8n} = -1 + 1 = 0$)]

27. (*b*) [L.C.M. $(a, b) =$ L.C.M.$(2^3 \times 3,\ 2 \times 3 \times 5)$

$= (2^3 \times 3 \times 5 = 8 \times 15 = 120$

L.C.M. $(a, b, c) =$ L.C.M. $(120, c)$

$=$ LCM $(120, 3^n \times 5)$

$=$ L.C.M. $(2 \times 2 \times 2 \times 3 \times 5,\ 3^n \times 5)$

$=$ L.C.M. $(2^3 \times 3 \times 5,\ 3^n \times 5)$

$= 2^3 \times 3^n \times 5 = 2^3 \times 3^2 \times 5$ (given) $\therefore$ $n = 2$]

28. (*c*) [$\sqrt{4} + \sqrt{9} = 2 + 3 = 5$]

29. (*c*) [$2 + \sqrt{5})\,(2 + \sqrt{5})$

$= 4 + 5 + 4\sqrt{5} = 9 + 4\sqrt{5}$]

30. (*a*) [$\left(\sqrt{2} - \sqrt{3}\right)\left(\sqrt{2} + \sqrt{3}\right) = 2 - 3 = -1$]

31. (*b*) [H.C.F. $(65, 117) =$ H.C.F. $(5 \times 13,\ 9 \times 13)$ $= 13$

$\therefore$ $65\text{ m} - 117 = 13 \Rightarrow 65\text{ m} = 130 \Rightarrow \text{m} = 2$]

32. (*a*) [$70 - 5 = 65$; $125 - 8 = 117$ and H.C.F. $(65, 117) = 13$]

33. (*d*) [$3 \nmid 10$, $3 \nmid 100$, $5 \nmid 504$]

34. (*c*) [$n^2 - 1 = (n - 1)\,(n + 1)$

$= (n - 1)\,((n - 1) + 2) = 0$ or 8, or 24, …]

35. (*b*) holds clearly.

36. (*b*) $3750 = 2 \times 3 \times 5 \times 5 \times 5 \times 5$

2	3750
3	1875
5	625
5	125
5	25
	5

$= 2 \times 3 \times 5^4$

$\therefore$ exponent of $5 = 4$

37. (*b*) $95 = 5 \times 19$; $171 = 9 \times 19$

$\therefore$ greatest possible speed $= 19$ m/minute

38. (*d*) holds clearly.

39. (*a*) $92 = 2 \times 2 \times 23 = 2^2 \times 23$

$152 = 2 \times 2 \times 2 \times 19 = 2^3 \times 19$

$\therefore$ HCF $(92, 152) = 2^2 = 4$

40. (*c*) $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

$25 = 5 \times 5 = 5^2$

$30 = 2 \times 3 \times 5 = 2^1 \times 3^1 \times 5^1$

Their L.C.M. $= 2^2 \times 3^1 \times 5^2 = 300$

$\therefore$ three alarm clock will next ring together after 300 minutes *i.e.*, after 5 hours.

i.e., at 5 pm.

41. (*a*) Since a and b are co-prime

$\therefore$ a^3 and b^3 are co-prime.

42. (*c*) $12 = 2^2 \times 3$; $21 = 3 \times 7$, $15 = 3 \times 5$

$\therefore$ HCF $= 3$ and LCM $= 2^2 \times 3 \times 7 \times 5 = 420$

43. (*c*) Since 3, 5, 15, 25 and 75 are the only common factors of 525, 3000.

$\therefore$ HCF = 75

44. (*c*) $2^5 \times 3^2$

45. (*b*) HCF $\times$ LCM = Product of two numbers

$$13 \times 40x = 65 \times 104$$

$$x = \frac{65 \times 104}{13 \times 40} = \frac{6760}{520} = 13$$

46. (*a*) $960 = 2^6 \times 3 \times 5$

$240 = 2^4 \times 3 \times 5$

$\therefore$ LCM (960, 240) $= 2^6 \times 3 \times 5$

$= 960$

47. (*d*) holds

48. (*b*) holds

49. (*c*) $7 \times 11 \times 13 + 5$

$1001 + 5$

$= 1006 = 2 \times 503$

= a composite number

Assertion and Reasoning Type Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as :

(a) **Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).**

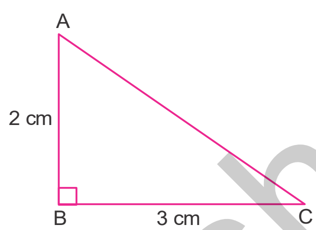
(b) **Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).**

(c) **Assertion (A) is true but Reason (R) is false.**

(d) **Assertion (A) is false but Reason (R) is true.**

1. **Assertion (A) :** The perimeter of ΔABC is a rational number.

Reason (R) : The sum of the squares of two rational numbers is always rational.



(C.B.S.E. 2024)

Sol. (***d***) Assertion (A) is false but Reason (R) is true.

2. **Assertion (A) :** If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM $(a, b, c) = 2^3 \times 3^2 \times 5$, then $n = 2$

Reason (R) : If LCM $(a, 18) = 36$ and H.C.F. $(a, 18) = 2$, then $a = 6$

Sol. Assertion (A) is clearly true

Reason (R) is false

$[\because a \times 18 = 36 \times 2 \Rightarrow a = 4]$

$\therefore$ (***c***) **holds.**

3. **Assertion (A) :** $\sqrt{8}$ is an irrational number.

Reason (R) : If m is a rational number, which is not a perfect square, then $\sqrt{m}$ is irrational.

Sol. Reason (R) is true [Standard Result]

Since 8 is not a perfect square $\therefore$ $\sqrt{8}$ is irrational

$\therefore$ Assertion (A) is true.

Since Reason (R) gives Assertion (A) $\therefore$ (***a***) **holds.**

4. **Assertion (A) :** 4^n, where $n \in$ N ends with the digit zero.

Reason (R) : If n is prime, then $\sqrt{n}$ is irrational.

Sol. Reason (R) is true [Standard Result]

Assertion (A) is false

$\because 4^n = (2 \times 2)^n = 2^n \times 2^n$

$\neq 2^m \times 5^n$ [m, n are +ve integers] $\therefore$ (***d***) **holds.**

5. **Assertion (A) :** $n^3 - n$ for all $n \in$ N, is divisible by 6.

Reason (R) : The product of three consecutive positive integers is divisible by 6.

Sol. Reason (R) is true [Standard Result]

Assertion (A) is true

$[\because n^3 - n = n(n^2 - 1) = n(n-1)(n+1)$

$= (n-1)\,n\,(n+1)]$

Since Reason (R) gives assertion (A)

$\therefore$ (***a***) **holds.**

6. **Assertion (A) :** H.C.F. (36 m^2, 18 m) = 18 m; where m is a prime number.

Reason (R) : H.C.F. of two numbers is always less than or equal to the smaller number. ***(C.B.S.E. 2026)***

Sol. Assertion (A) is true

Reason (R) is true

But (R) does not give (A)

$\therefore$ (***b***) **holds.**

Case Study / Source Based Integrated Questions

Case Study 1

To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

(*a*) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B ?
(*i*) 144 (*ii*) 128 (*iii*) 288 (*iv*) 272

(*b*) If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32 , 36) is
(*i*) 2 (*ii*) 4 (*iii*) 6 (*iv*) 8

(*c*) 36 can be expressed as a product of its primes as
(*i*) $2^2 \times 3^2$ (*ii*) $2^1 \times 3^3$ (*iii*) $2^3 \times 3^1$ (*iv*) $2^0 \times 3^0$

(*d*) $7 \times 11 \times 13 \times 15 + 15$ is a
(*i*) Prime number (*ii*) Composite number
(*iii*) Neither prime nor composite (*iv*) None of the above

(*e*) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a , b are prime numbers, then the LCM (p, q) is
(*i*) ab (*ii*) a^2b^2 (*iii*) a^3b^2 (*iv*) a^3b^3

(C.B.S.E. Q.B. 2021)

Sol. (*a*) Minimum number of books = LCM (32, 36)
$= 2^5 \times 3^2$
$= 32 \times 9 = 288$

$32 = 2^5$
$36 = 2^2 \times 3^2$

∴ **(*iii*) holds.**

(*b*) HCF (32, 36) × LCM (32, 36) = 32 × 36
⇒ HCF (32, 36) × 288 = 32 × 36

$$\Rightarrow \quad \text{HCF}(32, 36) = \frac{32 \times 36}{288} = \frac{36}{9} = 4$$

∴ **(*ii*) holds.**

(*c*)
$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

∴ $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$ ∴ **(*i*) holds.**

(*d*) $7 \times 11 \times 13 \times 15 + 15$ is a composite number
(∵ it is $15(7 \times 11 \times 13 + 1)$) ∴ **(*ii*) holds.**

(*e*) LCM (p, q) = LCM (ab^2, a^2b)
$= a^2 b^2$ ∴ **(*ii*) holds.**

Case Study 2

A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

(*a*) In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number participants that can be accommodated in each room are

(*i*) 14 (*ii*) 12 (*iii*) 16 (*iv*) 18

(*b*) What is the minimum number of rooms required during the event ?

(*i*) 11 (*ii*) 31 (*iii*) 41 (*iv*) 21

(*c*) The LCM of 60, 84 and 108 is

(*i*) 3780 (*ii*) 3680 (*iii*) 4780 (*iv*) 4680

(*d*) The product of HCF and LCM of 60, 84 and 108 is

(*i*) 55360 (*ii*) 35360 (*iii*) 45500 (*iv*) 45360

(*e*) 108 can be expressed as a product of its primes as

(*i*) $2^3 \times 3^2$ (*ii*) $2^3 \times 3^3$ (*iii*) $2^2 \times 3^2$ (*iv*) $2^2 \times 3^3$

(C.B.S.E. Q.B. 2021)

Sol. (*a*) Maximum number of participants = HCF (60, 84, 108) = 12 ∴ **(*ii*) holds.**

(*b*) Minimum number of rooms reqd. $= \frac{60}{12} + \frac{84}{12} + \frac{108}{12} = 5 + 7 + 9 = 21$ ∴ **(*iv*) holds.**

(*c*) LCM (60, 84, 108) $= 2^2 \times 3^3 \times 5 \times 7$
$= 4 \times 27 \times 35$
$= 108 \times 35$
$= 3780$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$
$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$
$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

∴ **(*i*) holds.**

(*d*) HCF × LCM = 12 × 3780 = 45360 ∴ **(*iv*) holds.**

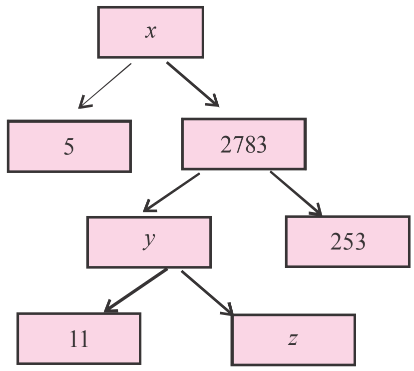
(*e*) $108 = 2^2 \times 3^3$ (from (*c*)) ∴ **(*iv*) holds.**

Case Study 3

A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.

Competency Based Questions

Observe the following factor tree and answer the following :



(a) What will be the value of x ?

(*i*) 15005 (*ii*) 13915 (*iii*) 56920 (*iv*) 17429

(b) What will be the value of y ?

(*i*) 23 (*ii*) 22 (*iii*) 11 (*iv*) 19

(c) What will be the value of z ?

(*i*) 22 (*ii*) 23 (*iii*) 17 (*iv*) 19

(d) According to Fundamental Theorem of Arithmetic 13915 is a

(*i*) Composite number (*ii*) Prime number

(*iii*) Neither prime nor composite (*iv*) Even number

(e) The prime factorisation of 13915 is

(*i*) $5 \times 11^3 \times 13^2$ (*ii*) $5 \times 11^3 \times 23^2$ (*iii*) $5 \times 11^2 \times 23$ (*iv*) $5 \times 11^2 \times 13^2$

(C.B.S.E. Q.B. 2021)

Sol. **(a)** $x = 5 \times 2783 = 13915$ $\therefore$ **(*ii*) holds.**

(b) $253 \times y = 2783 \Rightarrow y = \frac{2783}{253} = 11$ $\therefore$ **(*iii*) holds.**

(c) $11 \times z = 253 \Rightarrow z = 23$ $\therefore$ **(*ii*) holds.**

(d) 13915 is a composite number $[\because 13915 = 5 \times 11^2 \times 23]$ $\therefore$ **(*i*) holds.**

(e) (*iii*) **holds** [$\because$ of (*d*)]

5	13915
11	2783
11	253
	23

Section 1

THE FUNDAMENTAL THEOREM OF ARITHMETIC

Basic Concepts and Important Formulae

1. Every composite number can be expressed as a product of primes in ascending order and this decomposition is unique. [This is the Fundamental Theorem of Arithmetic.]
2. H.C.F. by Prime Factorisation or by Fundamental Theorem of Arithmetic.
   H.C.F. of two numbers a and b = Product of least power of common factors of a and b.
3. L.C.M. by Prime Factorisation Method.
   L.C.M. of two numbers a and b = Product of highest powers of all the factors of a and b.
4. Product of two numbers = (Their H.C.F.) × (Their L.C.M.)

1. State Fundamental Theorem of Arithmetic. *(CBSE 2016, 2020) (Remembering Type)*

Sol. Every composite number can be expressed as product of primes and this decomposition is unique, apart from the order in which the prime factors occur.

Thus if x is a composite number, then

$x = p_1 \, p_2 \, p_3 \, ... \, p_n$ is unique representation.

where $p_1, p_2 \, ... \, p_n$ are prime numbers in ascending order.

> **SIMILAR PROBLEM**
> The prime factorisation of a natural number is unique, each for the order of its factors.

2. Find the L.C.M. of 120 and 70 by fundamental theorem of arithmetic. *(CBSE 2010, 2013) (Understanding Type)*

Sol. $120 = 2^3 \times 3^1 \times 5^1$

$70 = 2^1 \times 5^1 \times 7^1$

$\therefore$ L.C.M. of (120, 70)

$= 2^3 \times 3^1 \times 5^1 \times 7^1$

$= 8 \times 105$

$= 840$

2	120
2	60
2	30
3	15
5	5
	1

2	70
5	35
7	7
	1

> **SIMILAR PROBLEM**
> Find the L.C.M of 96, 404
> **Ans.** 9696

3. Find the H.C.F. of 96 and 408 by prime factorisation method. Hence find their L.C.M. *(CBSE 2013) (Understanding Type)*

Sol. $96 = 2^5 \times 3^1$

$408 = 2^3 \times 3^1 \times 17^1$

H.C.F. of 96 and 408

$= 2^3 \times 3^1 = 8 \times 3 = 24$

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	408
2	204
2	102
3	51
17	17
	1

> **SIMILAR PROBLEM**
> Find the H.C.F of 306, 657 by prime factorisation method. Hence find their L.C.M.
> **Ans.** 22338

L.C.M. of 96 and 408 $= \dfrac{(96)\,(408)}{24} = (4)\,(408) = 1632$ $\quad \left[\because \text{L.C.M.} = \dfrac{\text{Product of numbers}}{\text{H.C.F.}}\right]$

4. If the H.C.F. of *a* and *b* is 12 and product of these numbers is 1800, then what is L.C.M. of these numbers. *(CBSE 2016) (Remembering Type)*

Sol. We know that

H.C.F. of two numbers × L.C.M. of the same numbers

= Product of these numbers.

$\therefore \quad 12 \times \text{L.C.M.} = ab = 1800 \quad \therefore \text{ L.C.M.} = \dfrac{1800}{12} = 150$

> **SIMILAR PROBLEM**
> If H.C.F. of 26, 91 is 13, find L.C.M. of 26, 91
> **Ans.** 182

5. The L.C.M. of two numbers is 14 times their H.C.F. The sum of L.C.M. and H.C.F. is 600. If one number is 280, find other number. *(CBSE 2012) (Application Type)*

Sol. $600 = \text{L.C.M.} + \text{H.C.F.} = 14\,(\text{H.C.F.}) + \text{H.C.F.}$

$= 15\,(\text{H.C.F.}) \quad [\because \text{L.C.M.} = 14\,(\text{H.C.F.})]$

$\therefore \quad$ H.C.F. $= \dfrac{600}{16} = 40$

Now $\quad$ L.C.M. = 14 (H.C.F.)

$= 14\,(40) = 560$

One number $= 280$

$\therefore$ (280) (Other number) = (H.C.F.) (L.C.M.) = (40) (560)

$\Rightarrow \quad$ Other number $= \dfrac{(40)\,(560)}{280} = (40)\,(2) = 80$

SIMILAR PROBLEM

L.C.M. of two numbers is 6 times their H.C.F. The sum of L.C.M. and H.C.F. = 91

If one number is 26, find other number.

(CBSE 2014)

Ans. 39

6. Using fundamental theorem of arithmetic, find the H.C.F. of 26, 52 and 91.

(CBSE 2012, 2011, 2014) (Application Type)

Sol.

$$26 = 2^1 \times 13^1$$
$$52 = 2^2 \times 13^1$$
$$91 = 7 \times 13^1$$

$$\begin{array}{r|l} 2 & 26 \\ \hline 13 & 13 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 52 \\ \hline 2 & 26 \\ \hline 13 & 13 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$\therefore$ H.C.F. (26, 52, 91) $= 2^1 \times 13^1 = 2 \times 13 = 26$

L.C.M. (26, 52, 91) $= 2^2 \times 13^1 \times 7^1$

$= 4 \times 91$

$= 364$

Note : $\quad 6 \times 72 \times 120 \neq$ H.C.F. (6,72, 12)

i.e., the product of three numbers is not equal to the product of their H.C.F. and L.C.M.

But $\quad$ L.C.M. $(p, q, r) = \dfrac{p.q.r.\,\text{H.C.F.}\,(p,q,r)}{\text{H.C.F.}\,(p,q)\,\text{H.C.F.}\,(q,r)\,\text{H.C.F.}\,(p,r)}$

H.C.F. $(p, q, r) = \dfrac{p.q.r.\,\text{L.C.M.}\,(p,q,r)}{\text{L.C.M.}\,(p,q)\,\text{L.C.M.}\,(q,r)\,\text{L.C.M.}\,(p,r)}$

7. If $n = (2^3)\,(3^4)\,(7)\,(15^6)$, then find the number of consecutive zeroes in natural number n.

(CBSE 2015, Application Type)

Sol.

$n = (2^3)\,(3^4)\,(7)\,(15^6)$

$= (2^3)\,(3^4)\,(7)\,(3^6)\,(5^6) \quad [\because 15 = 3 \times 5]$

$= (2^3)\,(5^3)\,(3^{10})\,(7)\,(5^3)$

$= (10)^3\,(3^{10})\,(7)\,(5^3)$

$= (1000)\,(3^{10})\,(7)\,(5^3)$

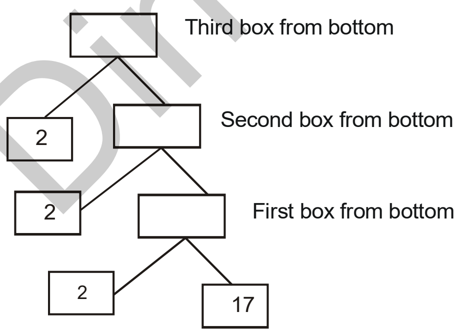
Thus n has three consecutive zeroes.

SIMILAR PROBLEM

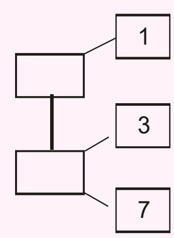
If $n = (2^4)\,(3^4), (7)\,(15^6)$, then find the number of consecutive zeroes in natural number n.

Ans. Four.

8. Find the missing number in the following factors used :



SIMILAR PROBLEM



Ans. 42, 2

(CBSE 2012, 2014) (Understanding Type)

Sol. The number in first box from bottom = (2) (17) = 34

The number in second box from bottom = 2 × 34 = 68

The number in third box from bottom = 2 × 68 = 136

9. **(*a*) Dhudnath has two vessels containing 720 ml. and 405 ml. of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.**
(CBSE 2015) (Understanding Type)

Sol. We have

$$405 = (5)(3^4)$$
$$720 = (5)(2^4)(3^2)$$

$\therefore$ H.C.F. of (405, 720)

$$= (5)(3^2)$$
$$= (5)(9)$$
$$= 45$$

$$\begin{array}{r|r} 3 & 405 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 720 \\ \hline 2 & 360 \\ \hline 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ in each glass 45 *ml* milk will be poured and minimum number of glasses

$$= \frac{720 + 405}{45} = \frac{1125}{45} = 25$$

SIMILAR PROBLEM

Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a tanker which can measure the petrol of either tanker in exact number of times. *(CBSE 2015)*

Ans. 170 litres.

9. **(*b*) The dimensions of a window are 156 cm × 216 cm. Arjun wants to put grill on the window creating complete squares of maximum size. Determine the side length of the square and hence find the number of squares formed.**
(CBSE 2026)

Sol. Dimensions of window are 156 cm × 216 cm

Side length of the square = HCF (156, 216)

$$156 = 2 \times 2 \times 3 \times 13$$
$$= 2^2 \times 3 \times 13$$
$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= 2^3 \times 3^3$$

HCF (156, 216) $= 2^2 \times 3 = 12$

$\therefore$ side length of the square = 12 cm

$$\therefore \text{ number of squares formed} = \frac{\text{Area of window}}{\text{Area of one square}}$$

$$= \frac{156 \times 216}{12 \times 12} = 13 \times 18 = 234$$

10. **Three alarm clock ring at intervals of 6, 9 and 15 minutes respectively. If they start ringing together, after what time will they next ring together ?**
(CBSE 2015) (Application Type)

Sol. We have

$$6 = (2)(3)$$
$$9 = (3)(3) = 3^2$$
$$15 = (3)(5)$$

$\therefore$ L.C.M. of (6, 9, 15) $= (2)(3^2)(5) = 90$

$\therefore$ three alarm clock will next ring together after 90 minutes.

SIMILAR PROBLEM

On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. Which is the minimum distance each shuld walk so that each can cover the same distance in complete steps ?
(NCERT Exemplar Problem)

Ans. 2520 cm

11. **Can two numbers have 18 as there H.C.F. and 380 as their L.C.M. ? Give reasons.** *(NCERT Exemplar Problem) (Application Type)*

Sol. H.C.F. = 18 (given)

L.C.M. = 380 Since 18 380

$\therefore$ H.C.F. of two numbers does not divide then L.C.M.

$\therefore$ given result is not true.

[$\because$ H.C.F. of two numbers always divides their L.C.M.]

SIMILAR PROBLEM

Can two numbers have 9 and 461 as H.C.F. and L.C.M respectively.
(NCERT Exemplar Problem)

Ans. No.

12. **Explain whether (3 × 5 × 13 × 46) + 23 is prime or composite number.**
(CBSE 2015) (Understanding Type)

Sol. $(3 \times 5 \times 13 \times 46) + 23$

$= (3 \times 5 \times 13 \times 2 + 1) \times 23$

$= 391 \times 23$, which is not prime number and hence is a composite number.

SIMILAR PROBLEM

Explain whether $5 \times 7 \times 11 \times 13 + 11$ is a prime number or composite number. *(CBSE 2014)*

13. **(*a*) Check whether 15^n can end with zero.** *(CBSE 2011, 2016) (Understanding Type)*

Sol. Let 15^n end with zero.

$\therefore$ 15^n is divisible by 10.

$\therefore$ it is divisible by 2.

$\therefore$ 15^n contains prime 2, which is not possible.

$[\therefore (15)^n = (3 \times 5)^n]$

$\therefore$ our supposition is wrong.

$\therefore$ $(15)^n$ cannot end with zero.

SIMILAR PROBLEM

(*i*) Show that 8^n cannot end with zero. ***(CBSE 2011, 2015)***

(*b*) **Show that 14^n cannot end with zero.** *(CBSE 2015) (Understanding Type)*

Sol. Let 14^n end with zero.

$\therefore$ 14^n is divisible by 10.

and $\therefore$ it is divisible by 5.

$\therefore$ 14^n contains prime number 5, which is not possible $[\because (14)^n = (2 \times 7)^n]$

$\therefore$ our supposition is wrong.

$\therefore$ $(14)^n$ cannot end with zero.

SIMILAR PROBLEM

(*i*) Check whether 6^n can end with the digit '5' for any natural number *n*.

(CBSE 2015, NCERT Exemplar, NCERT Book)

(*ii*) Check whether 12^n can and with the digit 0 or 5 for any natural number *n*. ***(CBSE 2015)***

14. **Find the H.C.F. of the smallest composite number and the smallest prime number.** *(CBSE 2013, 2018), (Remembering Type)*

Sol. The smallest composite number $= 2 \times 2 = 4$ and

The smallest prime number $= 2$

$\therefore$ H.C.F. between 4 and 2 = 2.

$(\because\ 4 = 2 \times 2 = 2^2\ ;\ 2 = 2^1)$

SIMILAR PROBLEM

What is H.C.F. of a pair of co-prime numbers.

Ans. 1.

15. **Find the LCM and HCF of 92 and 510, using prime factorisation.** *(CBSE 2023)*

Sol.

$$\begin{aligned} 92 &= 2 \times 2 \times 23 \\ 510 &= 2 \times 3 \times 5 \times 17 \\ \text{HCF} &= 2 \\ \text{LCM} &= 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460 \end{aligned}$$

16. **Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.** *(CBSE 2023)*

Sol. We have to find HCF of $85 - 1 = 84$ and $72 - 2 = 70$

HCF of 84 and 70 = 14

17. **Find the HCF of 45, 54, 270 using prime factorisation method.** *(CBSE 2024)*

Sol. Prime factorisation of

$$\begin{aligned} 45 &= 3 \times 3 \times 5 \\ 54 &= 2 \times 3 \times 3 \times 3 \\ 270 &= 2 \times 3 \times 3 \times 3 \times 5 \end{aligned}$$

$\therefore$ HCF (45, 54, 270) = 9

18. **Three bells toll at intervals of 9, 12 and 15 minutes respectively. If they start tolling together, after what time will they next toll together ?** *(CBSE 2024)*

Sol. Tolling together for next time means tolling after the least possible minutes which is the LCM of 9, 12 and 15.

$$\begin{array}{c|l} 2 & 9,\ 12,\ 15 \\ \hline 2 & 9,\ 6,\ 15 \\ \hline 3 & 9,\ 3,\ 15 \\ \hline 3 & 3,\ 1,\ 5 \\ \hline & 1,\ 1,\ 5 \end{array}$$

$\therefore$ LCM $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

$\therefore$ Time after which the three bells will toll together next = 180 minutes

We know that 60 minutes = 1 hour

$\therefore$ 180 minutes $= \dfrac{180}{60} = 3$ hours.

Subjective Questions

From N.C.E.R.T. Book / N.C.E.R.T. Exemplar Problems

19. Express 3825 as a product of its prime factors. *(NCERT Book)*

Sol.

$$\begin{array}{r|l} 3 & 3825 \\ \hline 3 & 1275 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$\therefore \quad 3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

20. (*a*) Find the L.C.M. and H.C.F. of 336 and 54 and verify that L.C.M. × H.C.F. = Product of two numbers. *(NCERT Book) (Remembering Type)*

Sol.

$$\begin{array}{r|l} 2 & 336 \\ \hline 2 & 168 \\ \hline 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{aligned} 336 &= 2 \times 2 \times 2 \times 2 \times 3 \times 7 \\ &= 2^4 \times 3 \times 7 \\ 54 &= 2 \times 3 \times 3 \times 3 = 2^1 \times 3^3 \\ \therefore \quad \text{L.C.M. } (336, 54) &= 2^4 \times 3^3 \times 7 \\ &= 16 \times 27 \times 7 \\ &= 3024 \\ \text{H.C.F. } (336, 54) &= 2^1 \times 3^1 = 6 \\ \therefore \quad \text{L.C.M.} \times \text{H.C.F.} &= 3024 \times 6 \\ &= 18144 \\ \text{Product of the numbers} &= 336 \times 54 \\ &= 18144 \end{aligned}$$

Thus L.C.M. × H.C.F. = Product of two numbers is verified.

20. (*b*) Find the H.C.F. and L.C.M. of 404 and 96 and verify that H.C.F. × L.C.M. = Product of two numbers. *(CBSE 2018)*

Sol.

$$\begin{array}{r|l} 2 & 404 \\ \hline 2 & 202 \\ \hline & 101 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r} 404 \\ 96 \\ \hline 2424 \\ 3636 \\ \hline 38784 \end{array}$$

$$\begin{aligned} \text{L.C.M. } (404, 96) &= 2^5 \times 3^1 \times 101 = 9696 \\ &[\because 404 = 2^2 \times 101 \text{ and } 96 = 2^5 \times 3] \\ \text{H.C.M. } (404, 96) &= 2^2 = 4 \\ \therefore \quad \text{H.C.F.} \times \text{L.C.M.} &= 4 \times 9696 = 38784 \\ \text{Product of 404 and 96} &= 404 \times 96 = 38784 \end{aligned}$$

Hence the result is verified.

21. Find the L.C.M. and H.C.F. of 12, 15, 21 by applying the prime factorisation method.

Sol.

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{aligned} 12 &= 2 \times 2 \times 3 = 2^2 \times 3 \\ 15 &= 3 \times 5 \\ 21 &= 3 \times 7 \\ \therefore \text{ L.C.M. } (12, 15, 21) &= 2^2 \times 3^1 \times 5 \times 7 \\ &= 60 \times 7 = 420 \\ \text{H.C.F. } (12, 15, 21) &= 3^1 = 3 \end{aligned}$$

22. Given that H.C.F. (306, 657) = 9, find L.C.M. (306, 657). *(NCERT Book) (Remembering Type)*

Sol. Since H.C.F. × L.C.M. of two numbers = Product of two numbers

$\therefore$ H.C.F (306, 657) × L.C.M. (306, 657) = 306 × 657

$$\therefore \quad 9 \times \text{L.C.M.} = 306 \times 657 \Rightarrow \text{L.C.M.} = \frac{306 \times 657}{9} = 306 \times 73 = 22338$$

23. Explain why 3 × 5 × 7 + 7 is a composite number.

(CBSE 2015) (NCER Exemplar Problem) (Understanding Type)

Sol. $3 \times 5 \times 7 + 7 = (3 \times 5 \times 1 + 1) \times 7 = (15 + 1) \times 7$

$= (15 + 1) \times 7 = 16 \times 7$, which is not a prime number. = a composite number.

24. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

(NCERT Book) (Understanding Type)

Sol. Consider $7 \times 11 \times 13 + 13 = (7 \times 11 \times 1 + 1) \times 13$

$= (77 + 1) \times 13$

$= 78 \times 13 = 6 \times 13 \times 13$

which is not prime, since it has 13 as one of the factors and 6 as one of the factors. (13, 6 are co-prime)

∴ it is a composite number.

Again $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5 = (1008 + 1) \times 5 = 1009 \times 5$ is not prime since it has 1009 and 5 as factors and they are coprime.

∴ it is a composite number.

> **SIMILAR PROBLEM**
>
> Explain why (5) (4) (3) (2) (1) + 5 is a composite number.

25. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point ?

(NCERT Book) (Remembering Type)

Sol. $18 = (2)(3)(3) = 2 \times 3^2$

$12 = (2)(2)(3) = 2^2 \times 3^1$

∴ L.C.M. (18, 12) $= 2^2 \times 3^2 = 4 \times 9 = 36$

∴ after 36 minutes, they will meet again at the starting point.

26. Find the H.C.F. and L.C.M. of 1530 and 2040. *(CBSE 2026) (Basic)*

Sol. $1530 = 2 \times 3 \times 3 \times 5 \times 17$

$= 2 \times 3^2 \times 5 \times 17$

2	1530
3	765
3	255
5	85
17	17
	1

$2040 = 2 \times 2 \times 2 \times 3 \times 5 \times 17$

$= 2^3 \times 3 \times 5 \times 17$

2	2040
2	1020
2	510
3	255
5	85
17	17
	1

∴ H.C.F. (1530, 2040)

$= 2 \times 3 \times 5 \times 17 = 30 \times 17$

$= 510$

LCM (1530, 2040)

$= 2^3 \times 3^2 \times 5 \times 17$

$= 8 \times 9 \times 5 \times 17$

$= 40 \times 153 = 6120$

Section 2

REVISITING IRRATIONAL NUMBERS

Basic Concepts and Important Formulae

1. **Rational Number**

   A number of the from $\frac{p}{q}$ where p, q are integers and $q \neq 0$ and $(p, q) = 1$ [*i.e.* there is no common factor between p, q other than 1] is called a **rational number** *i.e.* p, q are co-prime to each other.

2. **Irrational Number**

   A number which cannot be put in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$, $(p, q) = 1$, is called an **irrational number**.

   e.g. (*i*) Square root of a prime number is irrational. *e.g.* $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{11}$, etc. are all materials.

   (*ii*) Square root of a positive integer which is not a perfect square, is irrational. *e.g.* $\sqrt{2}, \sqrt{5}, \sqrt{10}, \sqrt{12}, \sqrt{14}, \sqrt{18}$ etc. are irrationals.

3. Rational number between 'a' and 'b'. $= \frac{a+b}{2}$ [where a, b are rational number]

   In fact between a and b, we can find an infinite number of rational numbers.

4. An irrational between 'a' and 'b' is $\sqrt{ab}$ if ab is not a perfect square.
   In fact between a and b, we can find an infinite number of irrational number.
5. Let p be a prime number. If p divides a^2, then p divides 'a' where a is a +ve integer.

Some Important Questions

FROM RECENT C.B.S.E. PAPERS

1. **Prove that $\sqrt{2}$ is irrational.** *(CBSE 2010, 2012, 2015, 2026 (Basic) NCERT Book) (Application Type)*

Sol. Let if possible, $\sqrt{2}$ be rational.

$\therefore \quad \sqrt{2} = \frac{r}{s}$, where r and s are integers and $s \neq 0$

Let r and s have some common factors other than 1.
Then divide r and s by that common factor and let us get

$\sqrt{2} = \frac{p}{q}$, where p and q are coprime and $q \neq 0$

$\Rightarrow \quad 2 = \frac{p^2}{q^2}$ [By squaring both sides]

$\Rightarrow \quad p^2 = 2q^2$ ... (1)

$\Rightarrow$ 2 divides p^2.

$\Rightarrow$ 2 divides p ... (2)

$\Rightarrow \quad p = 2k$, where k is an integer.

Putting $p = 2k$ in (1), we get

$4k^2 = 2q^2 \Rightarrow q^2 = 2k^2$

$\Rightarrow$ 2 divides q^2

$\Rightarrow$ 2 divides q ... (3)

SIMILAR PROBLEM

Prove that $\sqrt{3}$ is irrational.
(CBSE 2009, 2011, 2013)

Subjective Questions

From (2) and (3), we can say that p and q have a common factor 2, which is a contradiction to the fact that p and q are co-prime.

$\therefore$ our supposition that $\sqrt{2}$ is rational, is wrong.

Hence $\sqrt{2}$ is irrational.

2. Prove that $2\sqrt{2}$ is irrational. *(CBSE 2016) (Understanding Type)*

Sol. If possible, let

$2\sqrt{2}$ be a rational number.

$\therefore\ 2\sqrt{2} = \frac{p}{q}$, where p, q are integers and $q \neq 0$ and $(p, q) = 1$

$\Rightarrow\ \sqrt{2} = \frac{p}{2q} = \frac{\text{An integer}}{\text{non-zero integer}}$ $[\because q \neq 0, \Rightarrow 2q \neq 0]$

= a rational number $[\because (p, q) = 1 \Rightarrow (p, 2q) = 1]$

which is not possible. $[\because \sqrt{2}$ is an irrational number]

$\therefore$ our supposition is wrong.

Hence $2\sqrt{2}$ is irrational.

SIMILAR PROBLEM

(*i*) Prove that $\frac{5\sqrt{3}}{11}$ is irrational. ***(CBSE 2015)***

(*ii*) Prove that $3\sqrt{7}$ is irrational. ***(CBSE 2015)***

3. (*a*) Prove that $3 + 2\sqrt{5}$ is irrational. *(CBSE 2012, 2016, NCERT Book) (Application Type)*

Sol. If possible, let $3 + 2\sqrt{5}$ be rational.

$\therefore\quad 3 + 2\sqrt{5} = \frac{p}{q}$, where p, q are co-prime +ve integers and $q \neq 0$

$\Rightarrow\quad 2\sqrt{5} = \frac{p}{q} - 3 = \frac{p-3q}{q}$

$\Rightarrow\quad \sqrt{5} = \frac{p-3q}{2q}$ = rational number.

$\Rightarrow$ irrational number = a rational number, which is not possible.

$\therefore$ our supposition is wrong.

Hence $3 + 2\sqrt{5}$ is irrational.

SIMILAR PROBLEM

Prove that $2 + \sqrt{5}$ is an irrational number. ***(CBSE 2016)***

(*b*) Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number. *(CBSE 2018)*

Sol. Let if possible $5+3\sqrt{2}$ be rational

$\therefore\quad 5+3\sqrt{2} = r$ where r is rational.

$\therefore\quad 3\sqrt{2} = r - 5 \Rightarrow \sqrt{2} = \frac{r-5}{3}$ = a rational number

$\Rightarrow$ Irrational number = rational number which is not possible.

$\therefore$ our supposition is wrong. Hence $5+3\sqrt{2}$ is irrational.

4. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplication gives a rational or an irrational number. *(CBSE 2016)*

Sol.

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times 5}+3\sqrt{5\times 4}}{2\sqrt{5}} = \frac{(2\times 3)\sqrt{5}+3(2)\sqrt{5}}{2\sqrt{5}} = \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}} = \frac{12\sqrt{5}}{2\sqrt{5}}$$

= 6 = a rational number.

SIMILAR PROBLEM

Check whether $\frac{2\sqrt{405}+3\sqrt{80}}{4\sqrt{5}}$ on simplification gives a rational or an irrational number. ***(CBSE 2014)***

Ans. Rational.

Subjective Questions

5. Prove that $\frac{1}{2+\sqrt{3}}$ is an irrational number. *(C.B.S.E. 2010, 2015) (Application Type)*

SIMILAR PROBLEM

Prove that $\frac{1}{2-\sqrt{3}}$ is an irrational number.

(CBSE 2013)

Sol. $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

Let if possible $2 - \sqrt{3}$ be a rational number = r (say)

$\therefore \quad \sqrt{3} = 2 - r$ = (a rational number) [$\because$ difference of two rational numbers is a rational number]

which is not possible [$\because$ $\sqrt{3}$ is irrational]

$\therefore$ our supposition is wrong. Hence $\frac{1}{2+\sqrt{3}}$ is irrational.

6. Give an example to show that the product of a rational number and an irrational number may be a rational number.

(CBSE 2015)

SIMILAR PROBLEM

Give an example to show that the product of a rational number and an irrational number is irrational

Ans 2, $\sqrt{3}$

Sol. Take rational number = 0

$\therefore$ product of rational number and an irrational number

= (0) (irrational number) = 0 = rational

7. Give an example of two irrational numbers, whose

(*i*) sum is rational (*ii*) product is rational

SIMILAR PROBLEM

Give an example of two irrational number whose product is rational

Ans $\sqrt{2}, \sqrt{3}$

Sol. Consider two irrational numbers

$2 - \sqrt{3}$ and $2 + \sqrt{3}$

(*i*) Their sum $= 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

= a rational number

(*ii*) Their product $= (2 - \sqrt{3})(2 + \sqrt{3})$

$= (2)^2 - (\sqrt{3})^2$

$= 4 - 3 = 1$ = rational

SIMILAR PROBLEM

$\frac{6}{25}$ will terminate after how many places of decimal.

Ans. 2

8. Prove that $3 + 7\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. *(C.B.S.E. 2023)*

Sol. Let us assume that $3 + 7\sqrt{2}$ is a rational number.

$\Rightarrow 3 + 7\sqrt{2} = \frac{p}{q}$; where p, q are integers and $q \neq 0$

$\Rightarrow \sqrt{2} = \frac{p-3q}{7q}$

RHS is rational but LHS is irrational

$\therefore$ Our assumption is wrong

Hence, $3 + 7\sqrt{2}$ is an irrational number.

9. Prove that $\sqrt{5}$ is an irrational number. *(C.B.S.E. 2023,2026) (Standard)*

Sol. Let $\sqrt{5}$ be a rational number.

Subjective Questions

$\therefore \quad \sqrt{5} = \dfrac{p}{q}$, where $q \neq 0$ and p & q be co-primes.

$5q^2 = p^2 \Rightarrow p^2$ is divisible by $5 \Rightarrow p$ is divisible by 5.

$\Rightarrow \quad p = 5a$, where 'a' is some integer ...(i)

$25a^2 = 5q^2 \Rightarrow q^2 = 5a^2 \Rightarrow q^2$ is divisible by $5 \Rightarrow q$ is divisible by 5.

$\Rightarrow \quad q = 5b$, where 'b' is some integer ...(ii)

(i) and (ii) lead to contradiction as 'p' and 'q' are co-primes.

$\therefore \sqrt{5}$ is an irrational number.

10. Check whether the number 4^n can end with digit 0 for any natural number n. Give reasons for your answer. *(C.B.S.E. 2024)*

Sol. If the number 4^n, for any natural number n, were to end with the digit zero, then it should be divisible by 5. That is, prime factorisation of 4^n would contain the prime factor 5. But this is not possible because $(4^n) = (2^2)^n = 2^{2n}$, so, the only prime in the factorisation of 4^n is 2.

So, by uniqueness of the fundamental theorem of arithmetic, there are no other primes in the factorisation of 4^n.

Hence, there is no natural number 'n' for which 4^n ends with the digit zero.

11 Prove that $\dfrac{2-\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. *(C.B.S.E. 2024)*

Sol. Let us assume that $\dfrac{2-\sqrt{3}}{5}$ is rational. $\dfrac{2-\sqrt{3}}{5} = \dfrac{a}{b}$; $b \neq 0$ and a and b are integers.

$$\Rightarrow \quad 2b - \sqrt{3}b = 5a$$

$$\Rightarrow \quad -\sqrt{3}b = 5a - 2b$$

$$\Rightarrow \quad \sqrt{3} = \frac{2b-5a}{b}$$

Since a and b are integers so, $2b - 5a$ will also be an integer.

So, $\dfrac{2b-5a}{b}$ will be rational which means $\sqrt{3}$ is also rational.

But we know $\sqrt{3}$ is irrational (given)

Thus, a contradiction has risen because of incorrect assumption.

Thus, $\dfrac{2-\sqrt{3}}{5}$ is an irrational number.

> **SIMILAR PROBLEM**
>
> Prove that $14 - 2\sqrt{3}$ is an irrational number given that $\sqrt{3}$ is irrational.
>
> *(CBSE 2026)*

From N.C.E.R.T. Book / N.C.E.R.T. Exemplar Problems

12. Show that $5 - \sqrt{3}$ is irrational. *(NCERT Book) (Understanding Type)*

Sol. Let if possible $5 - \sqrt{3}$ be rational.

$\therefore \quad 5 - \sqrt{3} = \dfrac{p}{q}$, where p, q are integers and $q \neq 0$ and $(p, q) = 1$

$$\Rightarrow \quad \sqrt{3} = 5 - \frac{p}{q} = \frac{5q-p}{q}$$

Since p, q are integers

$\therefore\ 5q - p$ is an integer.

and $\because\ q \neq 0$

> **SIMILAR PROBLEM**
>
> Prove that $7 - 2\sqrt{3}$ is irrational.
>
> *(CBSE 2013, NCERT Exemplar Problem)*

Subjective Questions

$\therefore \quad \dfrac{5q-p}{q}$ is rational.

$\therefore \sqrt{3}$ is rational, which is not possible. [$\because$ 3 being prime $\Rightarrow \sqrt{3}$ is irrational]

$\therefore$ our supposition is wrong.

Hence $5-\sqrt{3}$ is irrational.

13. Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p, q are prime numbers.

(NCERT Exemplar Problem) (Application Type Problem)

Sol. Let if possible

$$\sqrt{p}+\sqrt{q} \text{ be rational.}$$

$$\therefore \quad \sqrt{p}+\sqrt{q} = r \text{ where } r \text{ is rational.}$$

$$\therefore \quad \sqrt{p} = r-\sqrt{q}$$

Sq. both sides, we get, $p = r^2+q-2r\sqrt{q}$

$$\Rightarrow \quad 2r\sqrt{q} = r^2+q-p$$

$$\Rightarrow \quad \sqrt{q} = \frac{r^2+q-p}{2r}$$

Now R.H.S. is a rational number. [$\because$ r is rational $\Rightarrow r^2$ is rational and p, q are integers *i.e.* rational]

$\therefore \quad$ L.H.S. $=\sqrt{q}$ is irrational [$\because$ q is prime]

$\therefore$ we arrive at a contradiction

$\therefore$ our supposition is wrong.

Hence $\sqrt{p}+\sqrt{q}$ is irrational.

SIMILAR PROBLEM

Prove that $\sqrt{3}+\sqrt{5}$ is irrational.

(NCERT Exemplar Problem)

(CBSE 2011)

Subjective Questions